

Free and Mixed Convection in Horizontal Porous Layers with Multiple Heat Sources

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Free and mixed convection has been studied numerically for horizontal porous layers heated from below by multiple, isothermal, discrete sources. Steady-state results have been obtained for natural convection with up to eight sources and for mixed convection with up to four sources. Rayleigh numbers are varied from 10 to 500, and Peclet numbers from 0.1 to 100. For natural convection, the vertical sidewalls for the two-dimensional domain considered have little effect on Nusselt numbers if the distance between the outermost source and wall is greater than three times the length of the heat source. As the number of heat sources increases, heat-transfer coefficients for the second outermost sources are a minimum. In mixed convection, overall Nusselt numbers increase with both the number of sources and Rayleigh number. If the individual heat source is considered, heat-transfer coefficients are determined only by its location. For Rayleigh numbers greater than 50, there exists a critical Peclet number for which the overall Nusselt number is a minimum. Unstable, oscillatory flow and temperature fields are observed for Rayleigh numbers greater than 50.

Nomenclature

- c = specific heat of fluid at constant pressure, J/kg·K
 D = length of the heat sources, m
 DN = n th discrete heat source
 d = distance between two discrete heat sources, m
 g = acceleration of gravity, m/s²
 H = height of the porous layer, m
 h = local heat-transfer coefficient, W/m²·K
 \bar{h} = average heat-transfer coefficient for the bottom surface, W/m²·K
 K = permeability, m²
 k = effective thermal conductivity of the saturated porous medium, W/mK
 N = number of the discrete heat sources
 Nu = local Nusselt number, hH/k
 Pe = Peclet number, $U_0 H/\alpha$
 Ra = Rayleigh number, $Kg\beta(T_h - T_c)H/\nu\alpha$
 T = temperature, K
 t = time, s
 U_0 = uniform velocity of the external flow, m/s
 x, y = Cartesian coordinates, m
 X = dimensionless distance on x axis, x/H
 Y = dimensionless distance on y axis, y/H
 α = effective thermal diffusivity of saturated porous medium, $k/(\rho c)_f$, m²/s
 β = coefficient of thermal expansion, 1/K
 θ = dimensionless temperature
 ρ = fluid density, kg/m³
 σ = heat capacity ratio of the saturated porous medium to that of the fluid, $[\phi(\rho c)_f + (1 - \phi)(\rho c)_s]/(\rho c)_f$
 τ = dimensionless time, $t/(\sigma H^2/\alpha)$
 ϕ = porosity
 ψ = stream function

Subscripts

- b = bottom surface
 c = cold surface
 f = fluid
 h = heated surface
 s = solid phase
 t = top surface

Introduction

OVER the past four decades, heat transfer in saturated porous media has received considerable attention for its important applications in geophysics and energy-related engineering problems. However, most previous studies of either natural or mixed convection have considered a horizontal porous layer uniformly heated from below, with an emphasis on the flow instabilities and restructuring.¹⁻⁶ Very few results have been reported for the case of a discrete heat source, although problems of this type are encountered more frequently in the applications.

Recently, Prasad and Kulacki^{7,8} and Lai et al.⁹⁻¹² have studied natural and mixed convection in horizontal porous layers partially heated from below. In these studies, extensive results have been presented for the case of a single heat source. However, results have not been reported for the case of multiple heat sources. In the present study, we numerically investigate the effects of multiple heat sources on the flow and temperature fields. Natural convection has been studied first, and its results are used as a limiting case for mixed convection. The problem under consideration has important applications in energy storage systems and nuclear waste repositories. In addition, one of the recent interests has been directed to the floor heating systems, which are widely used in European countries.

Formulation and Numerical Method

The geometry considered is a two-dimensional porous layer bounded by two horizontal impermeable walls (Fig. 1). The top wall is at a constant temperature T_c , and the rest of the walls other than the discrete heat sources are insulated. The porous layer is assumed to be fully saturated and has an initial temperature T_c . The discrete heat sources, which are assumed to have the same size and at a higher temperature T_h , are separated from each other by a constant interval. For mixed

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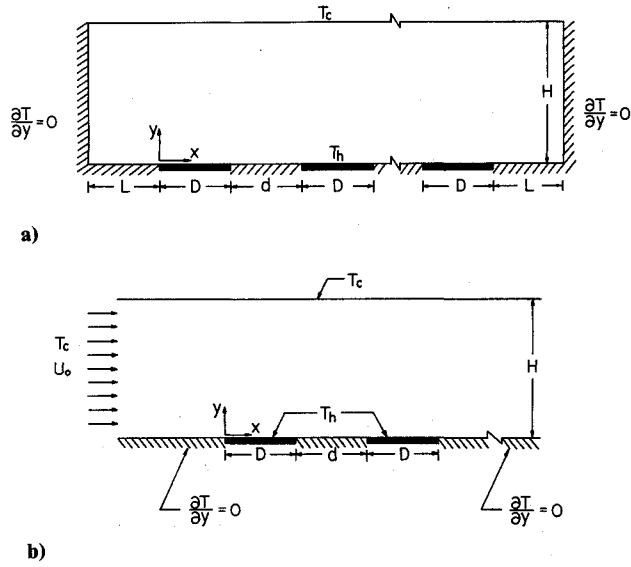


Fig. 1 A two-dimensional porous layer with multiple heat sources: coordinate system and boundary conditions: a) natural convection; b) mixed convection.

convection, a uniform flow at temperature T_c is introduced to the layer due to a hydrostatic pressure difference.

Having invoked the Boussinesq approximation, the governing equations based on Darcy's law, in terms of the stream function and dimensionless temperature, are given by

Natural convection:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \quad (1)$$

$$\frac{\partial \theta}{\partial \tau} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) - \left(\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} \right) \quad (2)$$

Mixed convection:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\frac{Ra}{Pe} \frac{\partial \theta}{\partial X} \quad (3)$$

$$\frac{\partial \theta}{\partial \tau} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) - Pe \left(\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} \right) \quad (4)$$

The transient forms of these equations are stated here for completeness. For the steady-state analysis, time derivatives are set to zero.

The corresponding boundary conditions are

Natural convection:

$$\psi = 0 \quad (5)$$

at all walls,

$$\theta = 0 \quad (6)$$

at the top wall,

$$\frac{\partial \theta}{\partial Y} = 0 \quad (7a)$$

at the bottom wall other than heat sources,

$$\theta = 1 \quad (7b)$$

at the heat sources, and

$$\frac{\partial \theta}{\partial X} = 0 \quad (8)$$

at the sidewalls.

Mixed convection:

$$\psi = Y, \quad \theta = 0, \quad X \ll 0 \quad (9)$$

$$\frac{\partial \psi}{\partial X} = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad X \gg 0 \quad (10)$$

Since a uniform external flow is taken as the upstream boundary condition for mixed convection, the stream function has the form of $\psi = Y + C$, where C is an arbitrary constant. For convenience, C can be assigned to 0 without the loss of generality. For the downstream boundary condition, Eq. (10), it is based on the fact that, far downstream from the heat sources, the flow will become parallel again after releasing a large portion of energy to the cooled wall. Axial conduction will then be negligible.

The governing equations are discretized using the control volume approach described by Patankar.¹³ An iterative scheme with under-/over-relaxation parameters is incorporated in the solution procedure to achieve fast convergence. The convective terms in the energy equation have been approximated by the QUICK scheme. This third-order-accurate scheme introduced by Leonard¹⁴ is preferred in the present study because it greatly reduced the numerical diffusion created by an upwind treatment and yet it avoided the problem of instabilities found with central differencing. The solution procedure has been discussed and successfully used in the study of mixed convection problems by the present authors.^{9,15} The details about the numerical scheme and the solution procedure are omitted here for brevity and may be found in Refs. 15 and 16.

To satisfy the boundary conditions of mixed convection, a sufficient length has been allowed in both upstream and downstream regions. The length of the unheated section in these regions is dependent on the Rayleigh and Peclet number. For a small Peclet number ($Pe < 1$), the flow and temperature fields basically retain the characteristics of natural convection; therefore, the upstream and downstream sections can be chosen to have the same length. For a higher Peclet number ($Pe \gg 1$), heat transfer is primarily by forced convection such that a sufficiently long downstream section is required to satisfy the boundary conditions. The consideration in selecting the length for the upstream and downstream regions has been discussed fully in Refs. 10 and 17 and will be omitted here for brevity. For the present study, it is found that the satisfaction of the boundary conditions is guaranteed if the upstream region is at least four times the length of the heat source and the downstream region is twice as long as the upstream section. Uniform grids have been used for the present study. A grid size of 0.05 is found to be satisfactory to obtain accurate results. For exploratory calculations, these grid fields predicted heat-transfer results within 3% of their asymptotic values and 2% for the maximum stream function. A variation of 10^{-4} or less in both ψ and θ at all nodes in the domain is the convergence criterion for the computation. In view of the parameters involved in this problem, we restrict our study to the case of $D = d = H$ and place our attention on the effects caused by the multiple heat sources.

Results and Discussion

Free Convection

As a preliminary study, the effect of wall confinement is investigated first since the results may apply to a porous layer of an infinite extent. These results are particularly helpful for implementing the calculations because savings from the computational effort and cost can be tremendous. The result of this preliminary study shows that the effects of the sidewalls are dependent on the Rayleigh number, but not the number of heat sources. For example, for $Ra \leq 100$, if the distance between the outermost heat source and the wall is greater than

twice the heater length, very little change occurs in the flow and temperature fields. In addition, Nusselt numbers (defined in the following section) are almost identical. However, for $Ra = 500$, these effects cannot be neglected unless $L \geq 3D$. Therefore, we can conclude that, for the case of a layer of infinite extent, accurate heat-transfer results can be obtained with a computational domain of $L \geq 3D$. In the following study on the effects of multiple sources on buoyancy-induced flow, the calculation domain is then chosen as $L = 4D$ to completely eliminate the effects of wall confinement.

A direct consequence of the existence of multiple heat sources in a porous layer is the generation of multiple pairs of recirculating cells and reversed thermal plumes (Figs. 2 and 3). Each additional heat source generates a pair of convective cells and one reversed thermal plume. The flow and temperature fields thus obtained are very similar to those reported by Prasad and Kulacki⁸ for a long single heat source. However, a close examination of these two cases reveals a basic difference. For a long single source, multiple convective cells only appear at a high Rayleigh number and $D \geq 3$. This can be clearly seen from Fig. 4, where the vertical velocity profiles are plotted for both cases. With multiple sources, the convective cells occur at a lower Rayleigh number because of the nonuniform heating. As the Rayleigh number increases, they exist for both cases.

The differences in the flow and temperature fields for these two cases can also be observed from the variations of the local Nusselt number, which are shown in Fig. 5. The Nusselt number is defined based on the local heat-transfer coefficient and is given by

$$Nu = \frac{hH}{k} = - \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0,1} \quad (11)$$

which also represents the local dimensionless heat flux. For a single source, when the Rayleigh number is small, heat transfer is mainly by conduction in a large portion of the domain directly above the source. As the Rayleigh number increases, recirculating flow starts, and heat transfer is then primarily by convection. For multiple sources, because of the nonuniform heating, recirculating flow can be initiated at a smaller Rayleigh number such that convection is always the mechanism for heat transport.

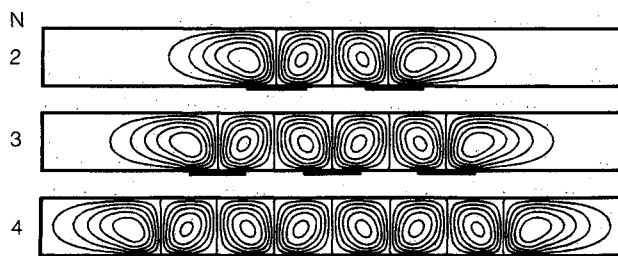


Fig. 2 Streamlines for natural convection over multiple heat sources; $Ra = 100$.

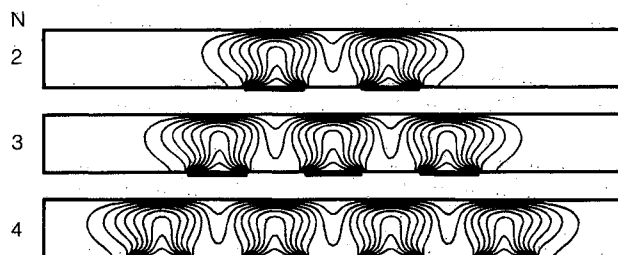


Fig. 3 Isotherms for natural convection over multiple heat sources; $Ra = 100$.

If the overall Nusselt number is based on the average heat-transfer coefficient, then

$$\begin{aligned} \overline{Nu} &= \frac{\bar{h}H}{k} = - \int \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} dX \\ &= - \sum_{i=1}^N \int_0^1 \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} dX \end{aligned} \quad (12)$$

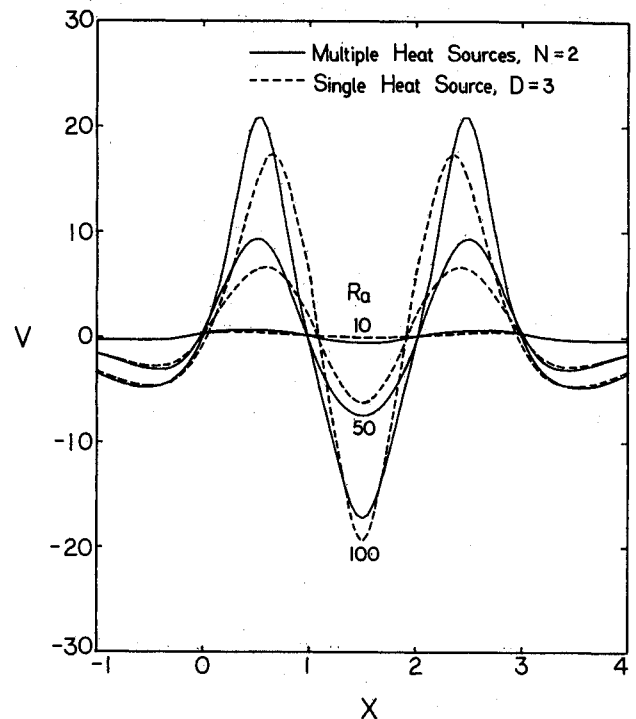


Fig. 4 Comparison of vertical velocity profiles at the midplane ($Y=0.5$); natural convection.

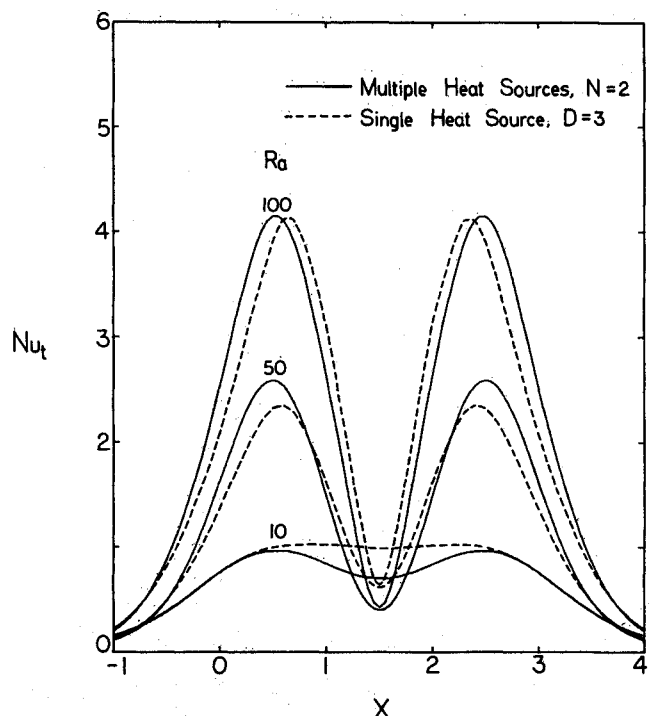


Fig. 5 Comparison of local Nusselt number at the top surface; natural convection.

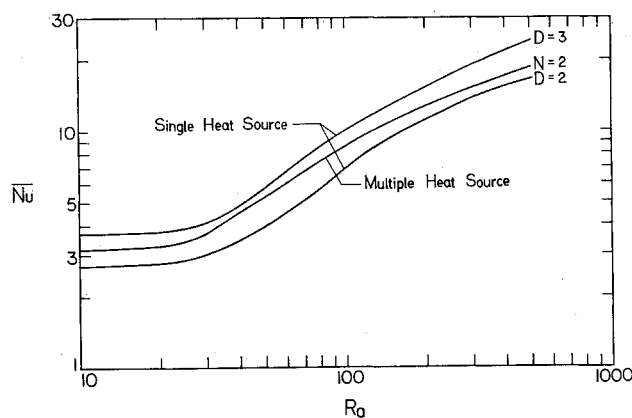


Fig. 6 Comparison of overall Nusselt numbers for natural convection in a porous layer with multiple heat sources.

which is the total dimensionless heat flux over the heat sources. The overall Nusselt number for $N=2$ is shown in Fig. 6. For comparison, overall Nusselt numbers for a single source with $D=2$ and 3 are also plotted in the same figure. It is interesting to note that the overall Nusselt number for $N=2$ is always greater than that for a single source of $D=2$, but is comparable to that of $D=3$. This is primarily dependent on the flow structure. As reported earlier,⁸ there exists only one pair of recirculating cells for a single source with $D=2$, whereas two pairs of cells exist when $N=2$. The increased recirculation due to these cells increases the overall Nusselt number even though the total area of the sources with $N=2$ is actually less than that for a single source with $D=3$.

For a fixed number of heat sources, the interaction between neighboring sources becomes more significant as the Rayleigh number increases. For a large Rayleigh number, the second outermost cells are suppressed considerably and pushed toward the center. As a result, isotherms are also twisted and move toward the center.

The instability in the flow and temperature fields^{18,19} has not been observed in the present study (at least, in the range of the parameters considered). Horne and O'Sullivan¹⁸ have concluded that oscillatory convection in the flowfield originates from combined processes of a "triggering mechanism" and the instability of the thermal boundary layer. As reported in an experimental study by the present authors,¹⁵ it is found that the triggering mechanism can be significantly weakened if one allows a sufficiently large unheated region in the layer, which provides additional capacity for dissipating small thermal disturbances generated in the heated region. This is apparent if one compares the geometry studied by the previous authors to the present case.

The overall Nusselt number for an individual heat source is listed in Table 1 and is observed to be independent of the total number of heat sources, but is determined only by its position in the field. Also, the overall Nusselt number for the second outermost source is a minimum, which is contrary to expectation. Normally, one would expect the Nusselt number for the source at the center to be a minimum. Confinement by the top wall and the interaction between neighboring sources appear to cause the location of this minimum. This can be verified by performing a test for which the height of the porous layer is

doubled. Results are shown in Table 2. Once the confinement by the upper boundary is removed, the overall Nusselt number for the second source increases and that of the center source becomes minimal.

The overall Nusselt number for the entire system is calculated by summing that for the individual ones. It increases with both Rayleigh number and the number of heat sources. However, for a fixed Rayleigh number, the contribution to the overall heat-transfer coefficient by an additional heat source becomes almost constant when $N>5$. For $N>5$, the heat sources in the central portion have almost the same values of overall Nusselt numbers. Therefore, for each additional heat source, the overall heat-transfer coefficient and heat transferred increase proportionately and by a constant increment.

Table 1 Overall Nusselt number for the individual heat sources

N	2		3		4	
Ra	$Nu(1,2)$	$Nu(1,3)$	$Nu(2)$	$Nu(1,4)$	$Nu(2,3)$	
10	1.5773	1.5780	2.4704	1.5780	1.4710	
20	1.6318	1.6330	1.4820	1.6330	1.4832	
30	1.8440	1.8463	1.6207	1.8463	1.6178	
50	2.6310	2.6350	2.3521	2.6350	2.3562	
80	3.7060	3.7112	3.3766	3.7112	3.3818	
100	4.2840	4.2906	3.9140	4.2904	3.9205	
200	6.2797	6.2972	5.6939	6.2968	5.7104	
300	7.5570	7.5845	6.7866	7.5834	6.8122	
500	9.2803	9.3276	8.2072	9.3258	8.2487	

N	5			6		
Ra	$Nu(1,5)$	$Nu(2,4)$	$Nu(3)$	$Nu(1,6)$	$Nu(2,5)$	$Nu(3,4)$
10	1.5780	1.4710	1.4715	1.5780	1.4710	1.4715
20	1.6330	1.4832	1.4845	1.6330	1.4832	1.4845
30	1.8463	1.6228	1.6250	1.8463	1.6228	1.6250
50	2.6350	2.3561	2.3603	2.6350	2.3561	2.3602
80	3.7112	3.3817	3.3870	3.7112	3.3817	3.3870
100	4.2904	3.9204	3.9270	4.2904	3.9204	3.9269
200	6.2968	5.7100	5.7271	6.2968	5.7104	5.7266
300	7.5835	6.8110	6.8380	7.5834	6.8112	6.8370
500	9.3258	8.2470	8.2905	9.3258	8.2470	8.2888

N	7			
Ra	$Nu(1,7)$	$Nu(2,6)$	$Nu(3,5)$	$Nu(4)$
10	1.5780	1.4710	1.4715	1.4715
20	1.6330	1.4832	1.4845	1.4845
30	1.8463	1.6228	1.6250	1.6250
50	2.6350	2.3561	2.3602	2.3602
80	3.7112	3.3817	3.3870	3.3869
100	4.2904	3.9204	3.9269	3.9268
200	6.2968	5.7100	5.7266	5.7262
300	7.5835	6.8110	6.8370	6.8358
500	9.3257	8.2470	8.2888	8.2870

N	8			
Ra	$Nu(1,8)$	$Nu(2,7)$	$Nu(3,6)$	$Nu(4,5)$
10	1.5780	1.4710	1.4715	1.4715
20	1.6330	1.4832	1.4845	1.4845
30	1.8463	1.6228	1.6250	1.6250
50	2.6350	2.3561	2.3602	2.3602
80	3.7112	3.3817	3.3870	3.3870
100	4.2904	3.9204	3.9269	3.9268
200	6.2968	5.7100	5.7266	5.7262
300	7.5835	6.8110	6.8370	6.8359
500	9.3257	8.2470	8.2888	8.2871

Table 2 Comparison of the overall Nusselt numbers for the individual heat sources ($Ra = 100$)

N	6			8			
	H	Nu(1,6)	Nu(2,5)	Nu(3,4)	Nu(1,8)	Nu(2,7)	Nu(3,6)
1	4.2904	3.9204	3.9269	4.2904	3.9204	3.9269	3.9268
2	8.0294	6.2736	6.2480	8.0294	6.2736	6.2468	6.2388

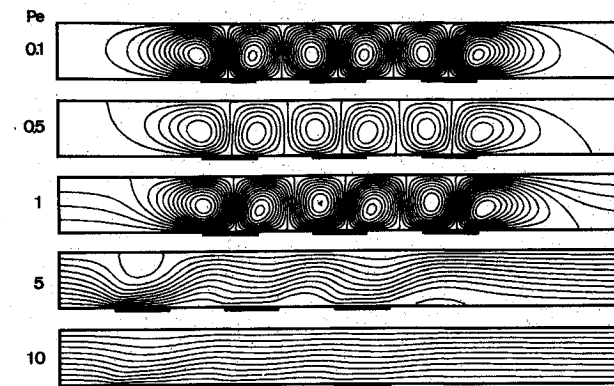


Fig. 7 Streamlines for mixed convection over three discrete heat sources.

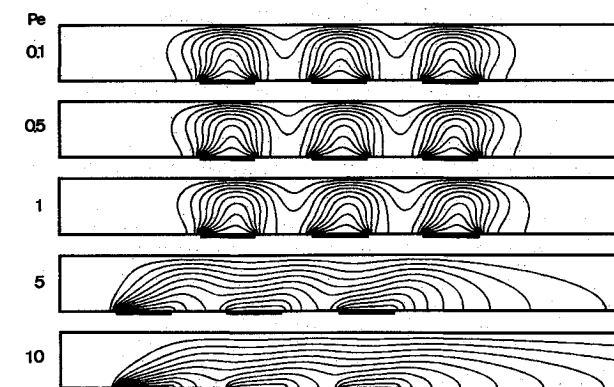


Fig. 8 Isotherms for mixed convection over three discrete heat sources.

Mixed Convection

If an external flow is introduced to the layer, the flow structure and the temperature field may be changed considerably, depending on the strength of the external flow (Figs. 7 and 8). When the Peclet number is small, buoyancy effects dominate, and the resulting flow and temperature fields are very similar to those observed for natural convection. As the Peclet number increases, the strength of recirculating cells is greatly weakened, and they die out eventually. It should be pointed out that Figs. 7 and 8 represent only a portion of the computational results in order to present a clearer picture of the region of maximum influence.

It is also interesting to note that, for mixed convection, the flow and temperature fields for multiple sources are very similar to those for a long, single source. Again, a close examination reveals a basic difference between them. At a small Peclet number, the situation is similar to that for the natural convection, which has been discussed earlier. However, as the Peclet number increases, the flow structure for these two cases is completely different, which can be observed in the vertical velocity profiles shown in Fig. 9. As seen, for a single heat source, the second inner cell (downstream side) is suppressed considerably, whereas the first one (upstream side) is expanded. For the multiple source case, both inner cells remain almost unaffected but are shifted downstream.

The effects of through flow on the temperature field can be seen in the variation of local Nusselt numbers (Fig. 10). As the Peclet number increases, the variation of local Nusselt numbers for a single source is totally different from that for the multiple source case, which is a result of the flow restructuring, as discussed earlier. For a single source, heat rejection to the top wall is mainly by the first inner cell, and the second cell has a less contribution, whereas for the multiple sources, both

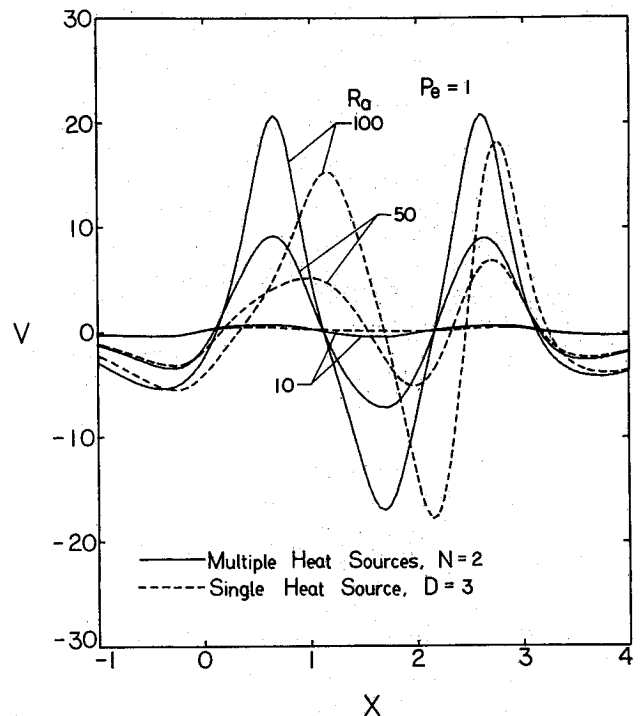


Fig. 9 Comparison of vertical velocity profiles at the midplane ($Y=0.5$): mixed convection.

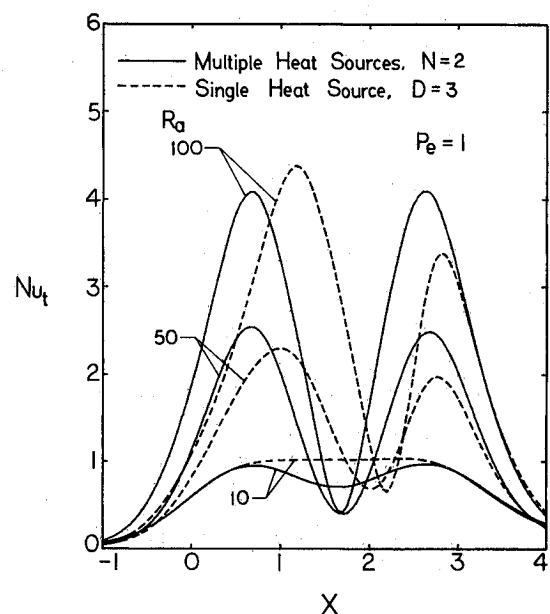


Fig. 10 Comparison of local Nusselt number at the top surface: mixed convection.

inner cells have equal contribution to the heat transferred.

Another interesting finding is that, for $Ra = 100$, the flow-field becomes unstable when $Pe \rightarrow 5$ (Figs. 11 and 12). A similar behavior has also been reported for the case of a long, single source.¹¹ By a transient analysis,¹² it has been shown that the instability of the flowfield is due to the destruction and regeneration of the recirculating flows. The mechanism that leads to his oscillatory behavior is similar to that which has been reported by Horne and O'Sullivan.¹⁸ The thermal disturbance generated by the leading source is carried downstream by the forced flow to initiate subsequent disturbances on the following sources. As a result of these repeated disturbances, the thermal boundary layer also becomes unstable. Clearly, the period of oscillation is a function of the Rayleigh and

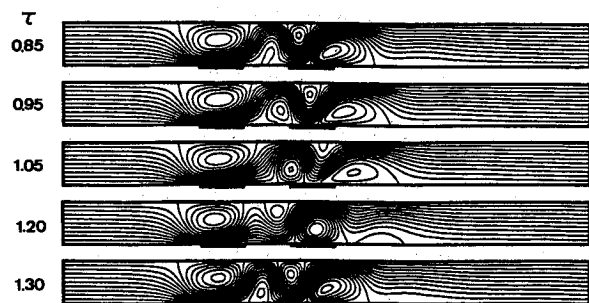


Fig. 11 Unstable flowfield for mixed convection over two discrete heat sources; $Ra = 100$ and $Pe = 5$.

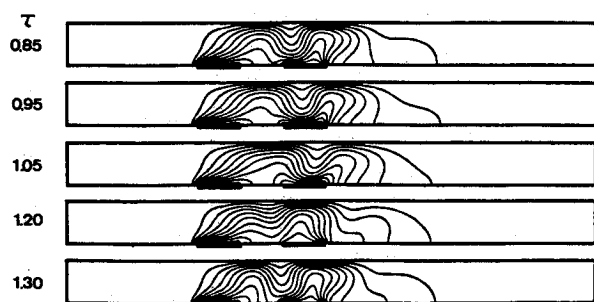


Fig. 12 Unstable temperature field for mixed convection over two discrete heat sources; $Ra = 100$ and $Pe = 5$.

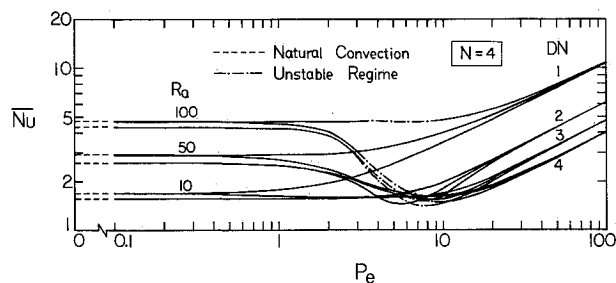


Fig. 13 Overall Nusselt number for individual heat source.

Peclet number, which represent the interaction between the buoyancy effects and the forced flow. For the case of $Ra = 100$ and $Pe = 5$, the period is estimated to be about 0.5 dimensionless time units.

The overall Nusselt number for the individual heat source is shown in Fig. 13 as a function of the Rayleigh and Peclet numbers. The overall Nusselt numbers for the corresponding natural convection case ($Pe = 0$) are shown as asymptotes in the same figure. For $Pe < 1$, it is found that the overall Nusselt numbers for mixed convection are not much different from those for natural convection. As the Peclet number increases, heat transferred by the leading source is enhanced considerably, whereas it is dramatically reduced for the following sources. This variation, which is particularly apparent at a large Rayleigh number, is due to the increased average fluid temperature as a consequence of more energy having been convected downstream, which then results in a reduction in the heat-transfer rate for the downstream heat sources. The overall Nusselt number for the second and following sources does not recover from their initial reduction until the Peclet number is increased further. When $Pe \approx 5$, the flow exhibits an oscillatory behavior as discussed earlier. At higher Peclet numbers, i.e., $Pe \geq 50$, Nusselt numbers approach values for the forced convection. This can be verified by examining the slope of

these curves, which gives 0.5, a value reported for the forced convection.

It is interesting to note that the Nusselt number for an individual source is independent of the total number of heat sources and is determined only by its position in the field with respect to the direction of the external flow. For example, the overall Nusselt number for the second source is only a function of Rayleigh and Peclet numbers, regardless of the total number of heat sources.

When the overall Nusselt number for the entire system is calculated, it is found that, for $Ra > 50$, there exists a critical Peclet number for which the overall Nusselt number is a minimum. As reported earlier,^{10,17} the existence of a critical Peclet number is attributed to the balance of the two driving forces. As the Peclet number increases, heat transfer is enhanced by forced convection; however, it is also reduced because of the diminishing effects of buoyancy. Therefore, a critical Peclet number exists only when the enhancement in heat transfer by the forced flow is offset by the reduction due to the diminishing buoyancy effects.

When compared to the overall Nusselt numbers for single source with $D = 2$ and 3, it is interesting to note that the overall Nusselt number for $N = 2$ is always greater than that for $N = 1$ and $D = 2$, but is comparable to the results when $D = 3$. This is strongly due to the difference in flow structure. As reported earlier,¹¹ there exists only one pair of recirculating cells in the flowfield for a single source when $D = 2$, whereas two pairs of cells are observed for $N = 2$. The enhancement of heat transfer via additional pairs of recirculating flows is quite obvious.

Conclusions

A numerical study has been conducted for free and mixed convection in horizontal porous layers locally heated from below with multiple, isothermal sources. For natural convection, multiple recirculating cells always exist in the flowfield. The enhancement of heat transfer by these recirculating cells is substantial, such that the overall Nusselt number for the multiple sources is always greater than that for a single source of the same size. For an individual heat source, the overall Nusselt number is independent of the total number of the sources but is determined only by its position in the field. It is found that the overall Nusselt number for the second source is a minimum for the present case ($H = D = d$). The overall heat-transfer coefficient increases with both Rayleigh number and the number of heat sources. When $N > 5$, it is found that the incremental contribution to the total heat transferred by adding an additional source becomes constant.

For mixed convection, the overall Nusselt numbers are not much different from those for the natural convection when the Peclet number is small. As the Peclet number increases, $Pe \geq 50$, the results approach those of the forced convection. For the individual heat source, the overall Nusselt number is also independent of the total number of the sources, but is determined only by its position in the field. For $Ra \geq 50$, a critical Peclet number exists for which the overall Nusselt number is a minimum. For $Ra = 100$, the flow becomes unstable when the Peclet number approaches 5. This flow instability is due to the instability of the thermal boundary layer.

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